ProCAST: A Projection Framework for Coupled Aggregation Constrained Multivariate Time Series Forecasting

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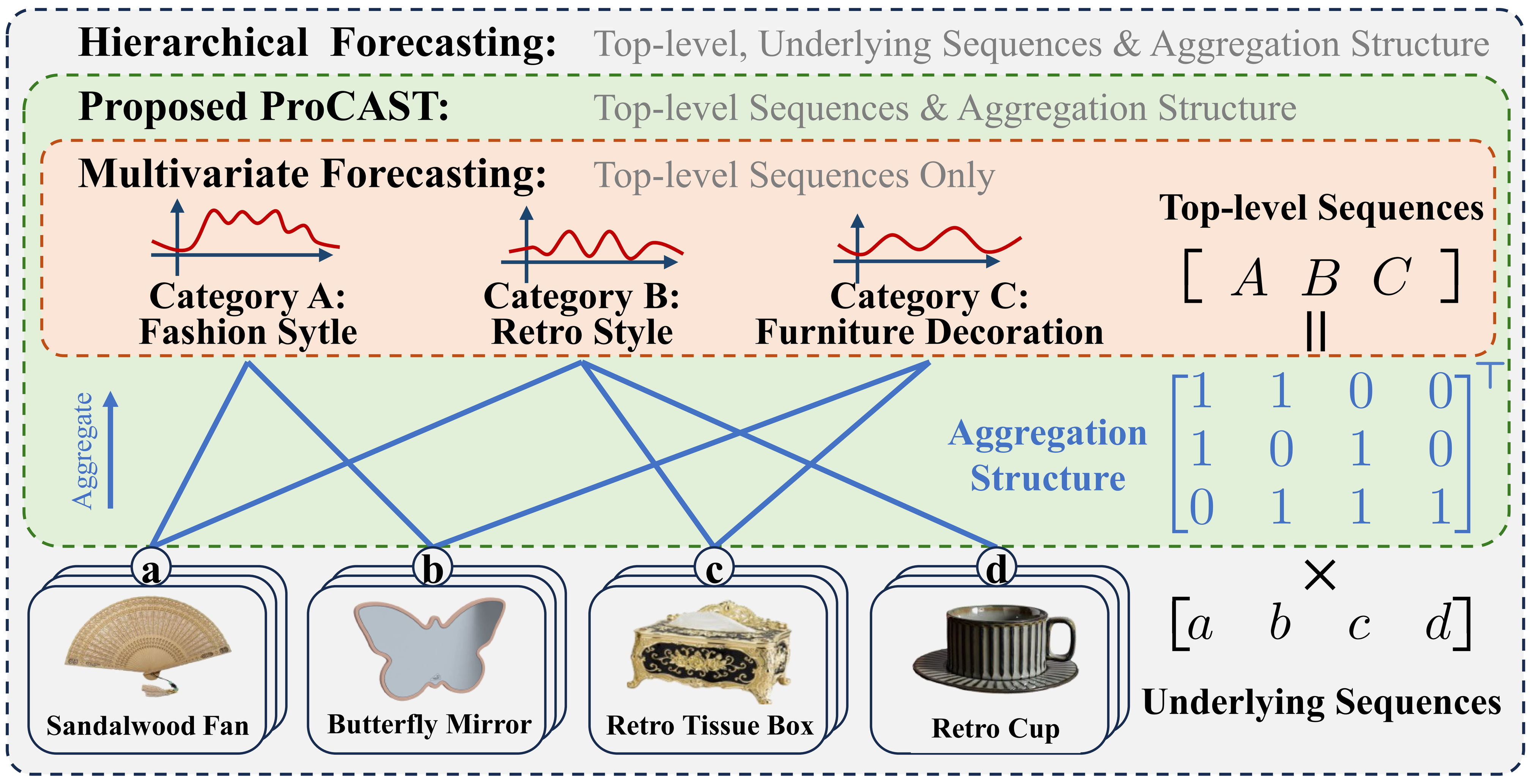
Introduction

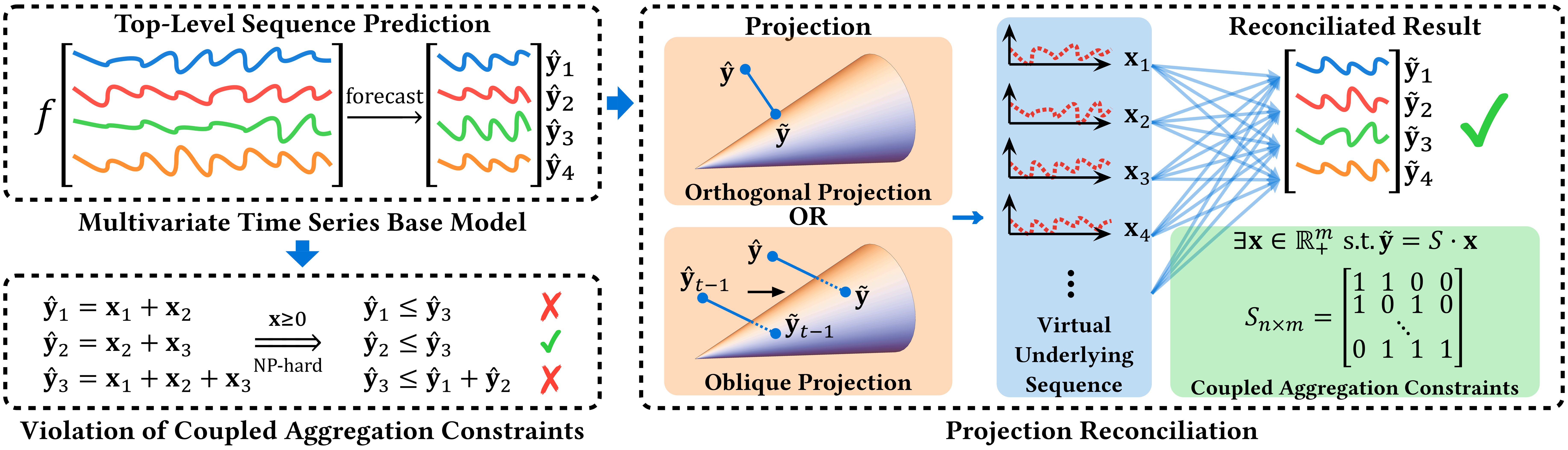
In aggregation structures, we often **focus on top-level forecasts**, as they are most relevant for sales trends, supply chain planning, and industry outlooks. When aggregation is coupled, simple non-negativity constraints on underlying sequences can **induce highly complex constraints on top-level sequences**.

Challenges ① constraints are intractable to derive; ② unconstrained forecasts rarely feasible; ③ underlying value often noisy/missing.

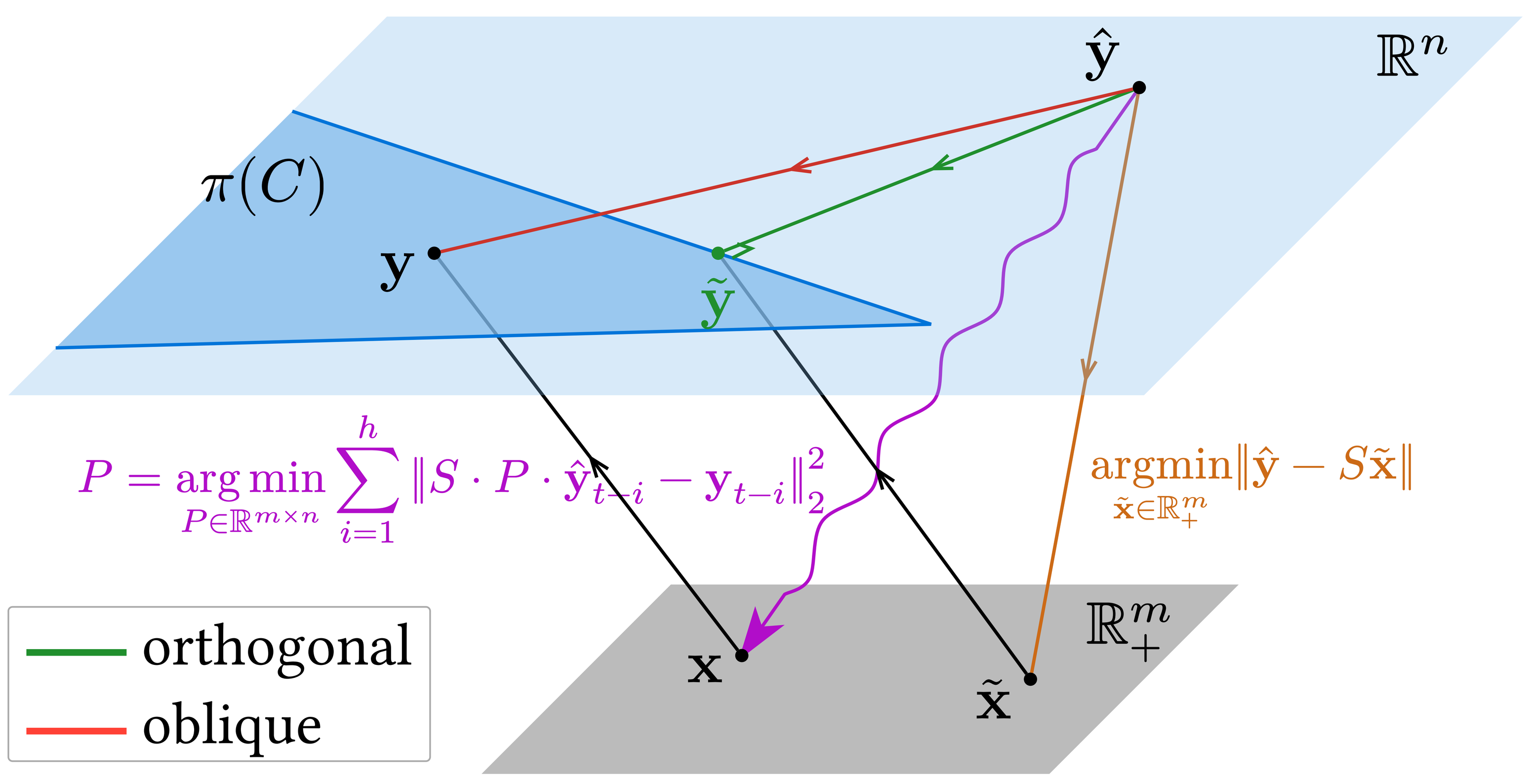
ProCAST integrates projection-based constraint enforcement into any base forecasting to deliver **accurate and feasible** top-level predictions with **theoretical guarantees**, while remaining effective without access to underlying sequences.



Overview of ProCAST, including two optional projection methods: orthogonal and oblique



Theoretical Guarantees



Orthogonal projection minimizes the Euclidean distance to the feasible set, **guaranteeing error reduction** under distance-based loss function.

$$\forall y \in \pi(C),\ \|y - \tilde{y}\|_2 \le \|y - \hat{y}\|_2$$

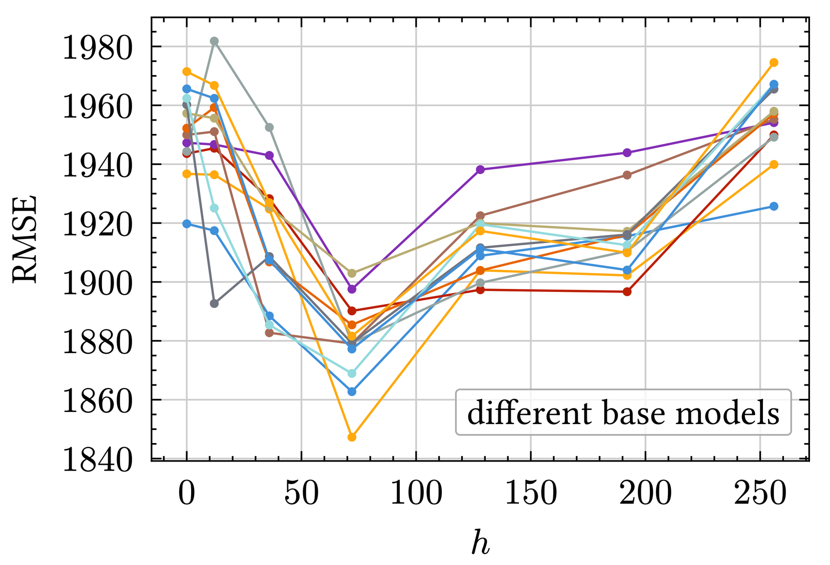
Oblique projection incorporate a data driven approach that learn a "projection matrix" from historical data, which enables the projected points to lie within the convex cone. Assuming stationarity over time $\tau \in [t-h, t]$, oblique projection **reduces the error in expectation**.

$$\mathbb{E}\left[\|\tilde{y}_\tau - y_\tau\|_2^2\right] \le \mathbb{E}\left[\|\hat{y}_\tau - y_\tau\|_2^2\right]$$

Experiment Results

Best results are **bolded**; second-best *underlined*; cells ~~shaded in gray~~ indicate predictions that violate Coupled-Coherence.

Dataset	E-Commerce Dataset										RH Dataset					
Metric	MAE					RMSE					MAE			RMSE		
Method	Raw	BU	MinT	Ortho.	Obliq.	Raw	BU	MinT	Ortho.	Obliq.	Raw	Ortho.	Obliq.	Raw	Ortho.	Obliq.
NBEATS	1092	1149	*1083*	1092	**1024**	1983	2052	*1975*	1983	**1877**	~~827.3~~	*827.3*	**752.7**	~~1262~~	*1262*	**1153**
NBEATSx	1092	1149	*1083*	1092	**1024**	1983	2052	*1975*	1983	**1877**	~~827.3~~	*827.3*	**752.7**	~~1262~~	*1262*	**1153**
NHITS	1103	1194	*1100*	1103	**1032**	*2001*	2102	2002	*2001*	**1901**	~~867.5~~	*867.5*	**807.0**	~~1331~~	*1331*	**1240**
TimesNet	*1050*	1175	1082	*1050*	**1038**	*1949*	2087	1989	*1949*	**1887**	~~*584.1*~~	584.6	**562.9**	~~880.4~~	*879.9*	**842.4**
TCN	*1067*	1258	1086	*1067*	**1054**	*1959*	2155	1985	*1959*	**1912**	~~*674.7*~~	674.7	**638.2**	~~1013~~	*1013*	**951.2**
BiTCN	*1073*	1174	1075	*1073*	**1069**	*1978*	2084	1979	*1978*	**1896**	~~3382~~	*3379*	**3046**	~~5674~~	*5655*	**5306**
DeepNPTS	*1058*	1209	1111	*1058*	**1039**	*1944*	2115	2015	*1944*	**1886**	~~664.0~~	*664.0*	**630.3**	~~1023~~	*1022*	**961.7**
TFT	*1065*	1236	1074	*1065*	**1048**	*1957*	2133	1969	*1957*	**1877**	~~*608.7*~~	610.2	**582.7**	~~913.7~~	*909.3*	**860.9**
TiDE	*1080*	1122	1120	*1080*	**1032**	*1985*	2033	2030	*1985*	**1886**	~~*1258*~~	1258	**1133**	~~1949~~	*1949*	**1762**
DLinear	~~1456~~	1549	1496	*1414*	**1319**	~~2411~~	2500	2406	*2330*	**2226**	~~3690~~	*3311*	**3283**	~~9510~~	*7620*	**7539**
Informer	3053	*2890*	3048	3053	**1406**	3702	*3560*	3698	3702	**2347**	~~2.2e4~~	*2.2e4*	**2.1e4**	~~2.7e4~~	*2.7e4*	**2.6e4**
Autoformer	**1050**	1114	1058	**1050**	*1052*	*1950*	2032	1963	*1950*	**1880**	~~1025~~	*1025*	**910.4**	~~1554~~	*1554*	**1388**
FEDformer	~~1053~~	1115	1059	*1053*	**1045**	~~1949~~	2029	1958	*1949*	**1874**	~~595.5~~	*595.5*	**571.9**	~~901.5~~	*901.4*	**854.7**
PatchTST	**1041**	1122	*1054*	**1041**	**1057**	*1924*	2030	1945	*1924*	**1881**	~~589.7~~	*589.6*	**561.8**	~~864.3~~	*864.3*	**817.9**
TimeXer	~~1074~~	1169	1082	*1074*	**1061**	~~1973~~	2075	1981	*1972*	**1897**	~~*799.3*~~	799.6	**741.3**	~~1214~~	*1213*	**1115**
TimeMixer	*1063*	1149	1067	*1063*	**1051**	*1966*	2062	1972	*1966*	**1887**	~~*608.3*~~	608.8	**585.9**	~~920.9~~	*920.5*	**878.1**
TSMixer	*1057*	1140	1065	*1057*	**1048**	*1959*	2054	1968	*1959*	**1877**	~~*619.0*~~	619.0	**589.0**	~~948.6~~	*948.6*	**888.6**
TSMixerx	*1062*	1134	1067	*1062*	**1057**	*1963*	2047	1966	*1963*	**1897**	~~*731.6*~~	733.5	**704.8**	~~1123~~	*1118*	**1061**
iTransformer	~~1097~~	1173	*1091*	1097	**1076**	~~1983~~	2075	*1981*	1982	**1928**	~~*612.7*~~	612.8	**595.2**	~~917.8~~	*916.4*	**883.7**
RMoK	*1072*	1162	1076	*1072*	**1047**	*1962*	2069	1969	*1962*	**1901**	~~*682.1*~~	682.6	**634.6**	~~1037~~	*1037*	**957.5**
SOFTS	~~1106~~	1174	*1093*	1106	**1091**	~~1984~~	2075	*1981*	1984	**1921**	~~*669.7*~~	670.5	**634.9**	~~986.8~~	*986.3*	**927.8**
StemGNN	*1059*	1236	1083	*1059*	**1055**	*1950*	2136	1981	*1950*	**1886**	~~*890.7*~~	894.0	**826.0**	~~1396~~	*1367*	**1258**



Results: Avg. RMSE reduction **5.15%** (E-Commerce) and **7.10%** (RH); MAE reduction **4.33%** and **6.19%** respectively. ProCAST removes all coupled-coherence violations and outperforms hierarchical baselines in practice. **Stationarity is consistently validated** across multiple base forecast models.

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Repo: https://github.com/HellOwhatAs/ProCAST/